JOURNAL OF COMPUTATIONAL PHYSICS 77, 330–339 (1988)

Artificial Cooling Due to Quiet Injection in Bounded Plasma Particle Simulations

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Received May 29, 1987; revised August 26, 1987

An explanation is proposed for an artificial cooling effect seen in electrostatic particle-in-cell plasma simulations. The effect hinges on heat transport from the trapped electrons to fluctuations of the electric field, which are kept at a sub-thermal level through the continuous "quiet" injection of passing electrons. Further simulations are done which test and support the explanation.

INTRODUCTION

Recently some simulations of a simplified single-ended Q-machine were performed in an operating regime in which solutions of the Vlasov equation indicated that a local potential maximum should occur, allowing electron trapping [1]. After a short initial transient, a near-equilibrium was reached and a significant number of trapped electrons were observed, with a distribution consistent with the Vlasov equation. Figure 1 shows the potential profile at this stage, and Fig. 2 shows the electron and ion phase spaces for the same time. Over a longer time (many thermal transit times), the trapped electrons both cooled (in thermal velocity) and increased in density. The width of the trapping region also increased (see Figs. 3 and 4). This unexpected cooling continued until the trapped-electron phase-space density was three to four times that of a Maxwellian distribution at the temperature of the passing electrons. This cooling violates the Vlasov equation and is an apparent violation of the second law of thermodynamics.

The Q-machine model is one-dimensional and electrostatic. Starting at $t = 0$ (the simulation region is initially empty), electrons and ions are injected from a hot plate (the left side of the system, which is chosen as $x = 0$) with a half-Maxwellian distribution, and particles which strike either side of the system are absorbed. In this case, the end plates are electrically isolated (an open external circuit), meaning that the boundary conditions on Poisson's equation are determined self-consistently from the amount of charge carried to (or from) the boundaries by the electrons and ions. The density of injected ions is chosen to be much larger than the density of

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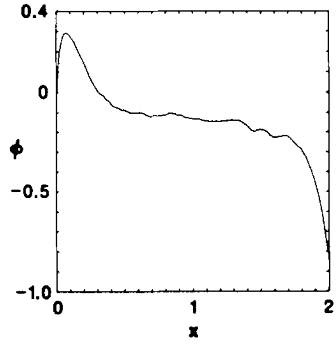

FIG. 1. Potential after initial transient, but before cooling of trapped electrons. This potential agrees with equilibrium theory.

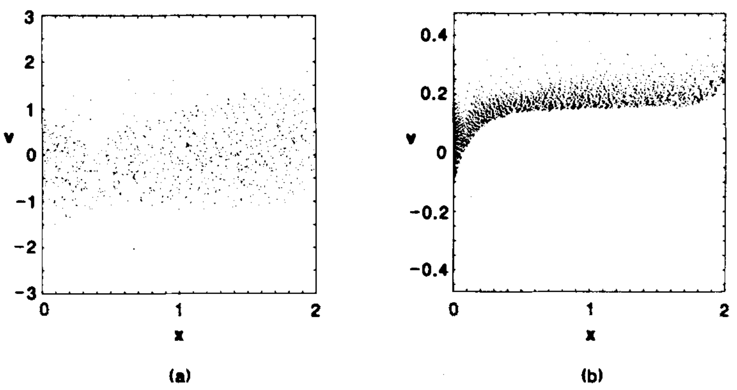

FIG. 2. Electron (a) and ion (b) phase spaces for potential in Fig. 1. Note smoothness and patterns due to quiet injection.

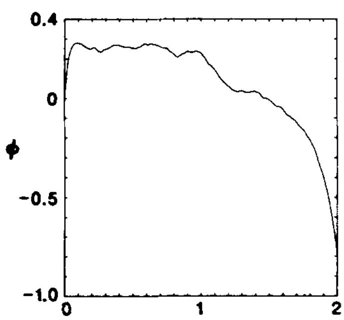

FIG. 3. Potential after some cooling of trapped electrons. Note elongation of trapping well.

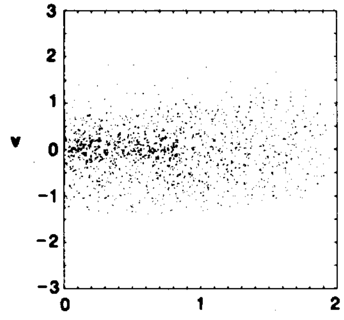

FIG. 4. Phase space for potential in Fig. 3. Note increased density of electrons in trapping well.

injected electrons, so that a potential maximum is created near the hot plate, thus creating a trapping well for electrons. The simulation parameters are shown in Table I.

No simple model predicts the observed cooling. The Vlasov model (fully time-dependent) predicts that the phase space density $f(x, v, t)$ in the trapped well may be no higher than the largest value of $f(x, v, t)$ at the point of injection ($x = 0$ for our simulations). For these simulations, this is also the value of f on the separatrix which defines the trapping region of phase space. Alternatively, one might expect that the distribution in the trapping well would fill in through some scattering process (due to the discrete particle model) to a Maxwellian shape. In fact, the distribution fills in far past this point.

TABLE I

Simulation Parameters

System length	2
Number of grid cells	256
Time step	1/128
Number of time steps	20,000
ε_0	1
q_e/m_e	-1
q_i/q_e	-1
m_i/m_e	40
v_{te}	1
v_{ti}	$1/\sqrt{40}$
Injected electron current	-63.83
Injected ion current	40.36
Injected electron flux	1277
Injected ion flux	807.4

Note. Units are arbitrary, but slef-consistent. Only dimensionless quantities are important.

Some possible explanations for this effect are: numerical inaccuracy, enhanced fluctuations due to an instability, and collisions. None of these explanations seem capable of accounting for the observed effect. The consideration of fluctuations, however, led to what seems to be the correct explanation.

An enhanced level of fluctuations would be expected to increase particle diffusion in velocity space, but in such a way as to *heat* trapped electrons. In thermodynamic equilibrium, this heating effect is balanced by a drag on the particles, giving rise to the Maxwellian distribution. The level of fluctuations in thermodynamic equilibrium is not zero, so an abnormally low level of fluctuations can be expected to *cool* electrons. Such a depressed level of fluctuations is in fact specifically introduced in order to increase the signal-to-noise level of most simulations. This artificially low level of fluctuations appears to be the cause of the artificial cooling observed in the Q-machine simulations.

Ruling Out Some Possible Causes

Three candidates for the cause of the cooling effect can be eliminated. These are numerical inaccuracy, fluctuations, and collisions.

Numerical inaccuracy was tested by changing the simulation parameters Δx and Δt, which represent the spatial grid spacing (which limits the spatial resolution), and the time step (which limits the temporal resolution). When one or both of these parameters was reduced (improving the resolution), the cooling effect remained with the same magnitude.

An enhanced level of fluctuations can be eliminated because they are a heating influence. Increasing the fluctuation level enhances diffusion in velocity in both directions, and so should create just as much flux out of the trapping well as into it. In fact, when the density of particles in phase space within the well is higher that outside it, one would expect more particles to leave the well than enter it.

Collisions are harder to eliminate. In one dimension the collision process is very different from that in three dimensions [2]. For instance, particles of the same species do not scatter when they collide, they either pass through each other or exchange velocities. Thus, the passing electrons cannot be cooling the trapped electrons through collisions in this simulation. Particles of differing species do not collide in the usual sense of introducing a random change in velocity, either. Their collisions exchange energy in a very simple and much more predictable way—either by reflecting velocities in the center of mass frame, or passing through each other with unaltered velocities. This has the result of greatly diminishing the effects of collisions relative to the three-dimensional case. Given these facts in combination it seems implausible that collisions could be responsible for the effect. This is admittedly a weak argument; the strongest evidence for collisions being unimportant is the simulation evidence presented here for the depressed level of fluctuations being the only process of importance.

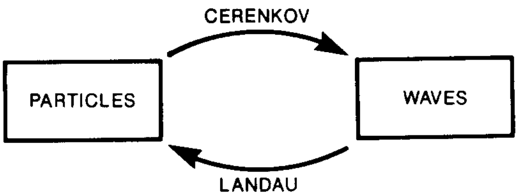

FIG. 5. Energy balance between particle thermal energy and wave fluctuations in periodic or infinite plasma.

ELECTRIC FIELD FLUCTUATIONS

Thermal energy may reside in one of two reservoirs in a plasma: disorganized particle motion and fluctuations of the electrostatic wave field. The word field is used here not in the sense of the electric field, but in the sense of the set of waves which are supported by a medium (as in quantum field theory). It is necessary to consider the electrostatic wave field as being something separate from the electric field, since the *organized* motion of particles plays a role in the electrostatic waves in the plasma. At short wavelengths the distinction between wave fluctuations and particle fluctuations becomes blurry, but at long wavelengths the distinction is clear.

There is coupling between these reservoirs, and energy flows between the two. In thermodynamic equilibrium, the flow of energy from wave fluctuations to particles is exactly balanced by the flow of energy from particles to wave fluctuations. These flows are called respectively Landau damping and Cerenkov emission, and the flow rates are proportional to the energy present in the reservoir which is losing energy (see Fig. 5). The balance between these flows in thermodynamic equilibrium gives rise to the fluctuation–dissipation theorem, which relates the temperature to the level of electric field fluctuations in the plasma.

QUIET INJECTION

In order to reduce the background noise level in particle simulations, particles are usually put into phase space (either loaded initially or injected over time) nearly uniformly. In the present simulations a "bit-reverse" scheme was used. This has the effect of drastically reducing the amplitude of fluctuations in the electric field (see Chap. 16 of [3] for a description of the bit-reverse scheme and its effect). For computer runs of short duration, this is entirely beneficial. Even for long runs, if the boundary conditions are periodic, the worst that can happen is that the fluctuation level of the electric field ($\langle E^2 \rangle$) rises up to the natural level, and since the total heat content of the fluctuations is not large (i.e., $\varepsilon_0 \langle E^2 \rangle / nkT$ is small), and the system is closed, the particles do not cool appreciably (see Fig. 6). This cooling of

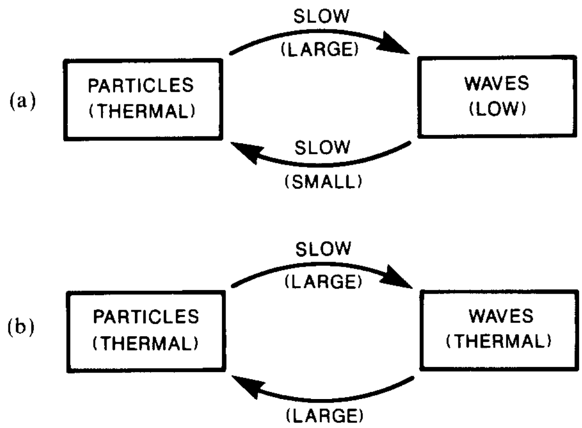

FIG. 6. Transition from initial quiet start (a) to final equilibrium (b) takes place on a slow time scale. The final energy content of the waves is much smaller than that of the particles.

particles in periodic simulations has not been explicitly reported, but it must occur and should be observable. (Simulation experts may be reminded of Gitomer and Adam [4], but the effect they noted was a *rapid* rise in the level of fluctuations due to a multibeam instability—a very different phenomenon from that described here.)

A bounded simulation is an open system, and the situation is quite different. Most particles are in the system only for one or two transit times. Only the particles which are trapped stay for long times. The wave fluctuations, however, are not trapped at all. Since they are collective modes, they move at their group velocities (which are strongly affected by the bulk motion of the plasma). Thus the wave fluc-

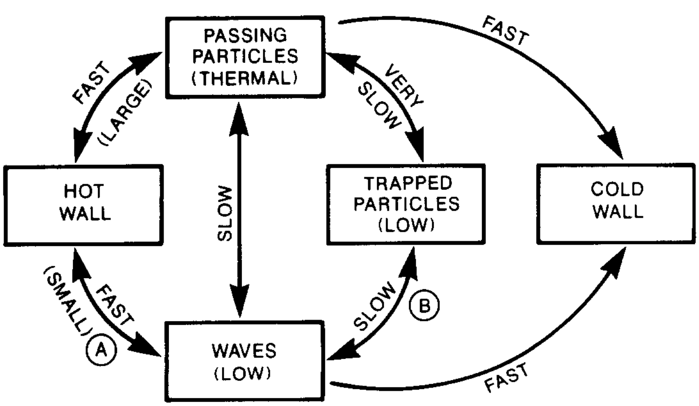

FIG. 7. Wave fluctuations equilibrate on a fast time scale to a low level (due to low level of injection noise) (A). Trapped electrons equilibrate on a slow time scale to low level by losing energy to low temperature fluctuations (B).

tuations can and do leave the system, only to be replaced with the artificially reduced fluctuations in the newly injected particles. In other words, the trapped electrons can cool via Cerenkov emission of waves which then leave the system. The energy flow is now much more complicated than in the periodic case (see Fig. 7). This mechanism is completely physical; if the noise level of injected electrons could be reduced in the laboratory, this effect would occur.

SIMULATION EVIDENCE

Simulation offers many methods of testing the hypothesis that the quiet injection of particles is responsible for the cooling of the trapped electrons. Several of these were tried, and all supported the hypothesis.

Figure 8 shows the results of a quiet start run. Plotted are the electron velocity distribution function averaged over a narrow region near the potential maximum at the end of the run, and the time history of the total number of electrons in the system. Note the elevated electron density and the decreased temperature of the trapped electrons in the distribution function plot, and the steady increase in the total number of electrons.

The most obvious test of the hypothesis is to replace the quiet injection scheme with one which has full noise, meaning that the times of injection are random with a uniform distribution in time, and that the velocities of injection are random with a half-Maxwellian distribution. The result, shown in Fig. 9, is that the system settles down to an equilibrium very quickly, and the trapped electrons never cool below the temperature of injection.

One more piece of information can be inferred from this run. A valid objection to the support this run gives the hypothesis is that *any* source of noise will cause

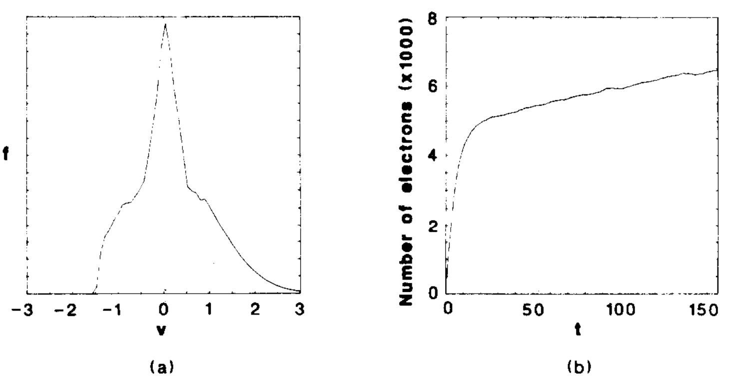

FIG. 8. Electron distribution at position of trapping well at end of run (a), and time history of the total number of electrons in the system (b) for quiet injection. The number of electrons grows as more electrons become trapped.

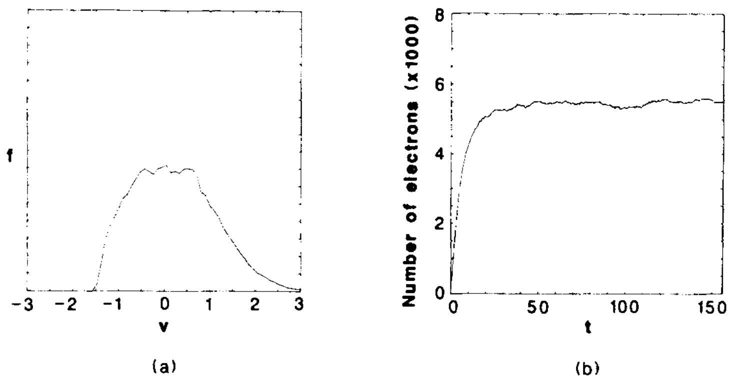

FIG. 9. Electron distribution at position of trapping well at end of run (a), and time history of the total number of electrons in the system (b) for noisy injection.

heating, and so it is not obvious that the correct source of noise has been found or that lack of noise is indeed responsible for the effect. This run shows, however, that the noise is of roughly the right magnitude. If the noise level were too high, then the trapped electrons in the noisy case would reach a Maxwellian profile sooner than the quiet case (which initially comes to the flat distribution predicted by Vlasov theory), since fluctuations enhance diffusion in *both* directions across the separatrix. If the noise level were too low, some residual cooling effect would be seen. That neither of these effects are seen lends support for the hypothesis.

To further test the hypothesis, some runs were made with four times as many particles. Since the physical parameters were kept constant, the charge and mass of each particle was decreased by a factor of four. In the quiet start case, since the

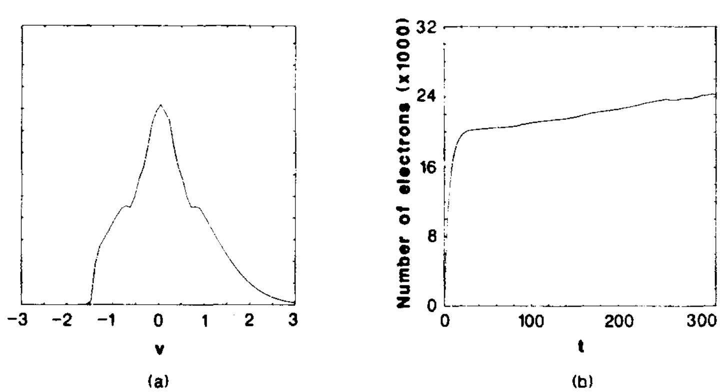

FIG. 10. Quiet injection results for electron distribution (a) and total number of electrons (b) with four times as many particles as in Fig. 8. (Note that this simulation was also run for twice as long as in Fig. 8.)

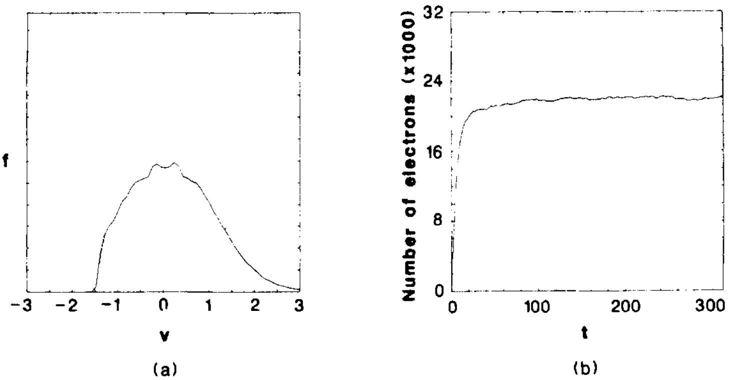

FIG. 11. Noisy injection results for electron distribution (a) and total number of electrons (b) with four times as many particles as in Fig. 8.

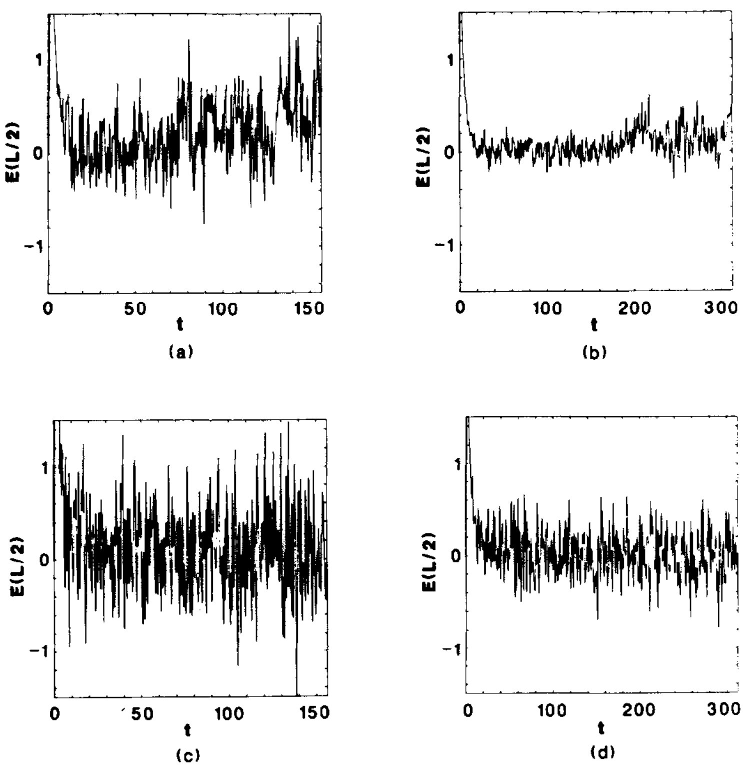

FIG. 12. Time histories of electric field in center of simulation region for four different runs. (a) and (b) are quiet injection runs with (b) having four times as many particles as (a), and (c) and (d) are noisy injection runs with (d) having four times as many particles as (c). RMS value of E in (b) should be $\frac{1}{4}$ that of (a), and RMS value of E in (d) should be half that of (c).

electrons are injected quasi-regularly, the fluctuations in the charge density should vary inversely with the number of particles, in this case a factor of four. This in turn should result in a factor of four reduction in the RMS fluctuation of the electric field $\langle (E-\bar{E})^2 \rangle^{1/2}$. The diffusion rate is proportional to $\langle (E-\bar{E})^2 \rangle$, so the rate at which each particle diffuses into the trapping well would be reduced by a factor of 16. (Since there are four times as many particles, the absolute *number* of particles diffusing into the trapping well would be reduced by a factor of four.) This is *not* observed.

In the case of full thermodynamic noise, the fluctuation–dissipation theorem dictates that increasing the number of particles by a factor of four must result in the mean-square electric field fluctuation $\langle (E-\bar{E})^2 \rangle$ decreasing by a factor of four. This in turn should reduce the rate of diffusion by only a factor of four. (In this case, the absolute *number* of particles diffusing into the trapping well in a given time interval will not be reduced.) As Figs. 10 and 11 show, the quiet and noisy runs obey roughly the expected behavior with the diffusion being due to the *full* thermodynamic noise of the trapped particles. The electric field at the mid-plane was also followed to see if it obeys the behavior described in the last paragraph, and as Fig. 12 shows, it does.

Conclusion

Simulations support the hypothesis that quiet loading in bounded plasma particle simulations causes artificial cooling of trapped electrons. This effect is very small, and therefore has not been reported, in periodic simulations because of the low heat capacity of the wave fluctuation field, but it will be a limitation on the accuracy of some bounded plasma simulations.

Acknowledgments

I gratefully acknowledge the help of Professors C. K. Birdsall and S. Kuhn, as well as Dr. T. L. Crystal and P. G. Gray. This research was sponsored by Department of Energy contract DE-AT03-76ET53064 and Office of Naval Research Contract N00014-85-K-0809. The computer simulations were performed on the CRAY computers of the National Magnetic Fusion Energy Computer Center.

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